REVIEW ARTICLE

THE TENDENCY TO GENERALIZE: A FEATURE OF LATE ANTIQUE AND MEDIEVAL MATHEMATICS, OR A FLAW IN MODERN HISTORIOGRAPHY?*

The title of Netz's book promises a wide-ranging study on the transformation of mathematics from the third century B.C.E. (Archimedes' time) to the eleventh century C.E. (al-Khayyam's time). It is actually in no way an exhaustive study of the subject; it rather focuses on the solutions found during this long span of time to a particular geometrical problem. As we shall see, this particular study is given a paradigmatic value, hence the title.

The paradigmatic case is presented in the first of the three parts of N.'s book, entitled "The Problem in the World of Archimedes." In it, N. comments on various solutions found in early Hellenistic time to the famous fourth geometrical problem listed by Archimedes in *SC* 2.4:[1] "to cut a given sphere, so that its segments have to each other a given ratio" (Pb1).[2] Archimedes' solution consists in a long and complex reduction,[3] through the ancient technique of analysis, to a more general problem, which is the following: "to cut a given line ΔZ at X, so that we should have the following proportion: the one segment XZ should be to some given line as some given area is to the square on the other segment ΔX" (Pb2).[4] N. very quickly discusses Archimedes' solution (part 1.1, pp. 11–13), and then goes on to propose a translation of what the sixth-century C.E. commentator Eutocius says (with N.'s approval, as we shall see) was Archimedes' solution to Pb2 (part 1.2).[5] N. goes on

* *The Transformation of Mathematics in the Early Mediterranean World: From Problems to Equations.* By Reviel Netz. Cambridge: Cambridge University Press, 2004. Pp. [ix] + 198. £50.00, $70 (hardback).

1. Abbreviations: *SC* = Archimedes' treatise *On the Sphere and the Cylinder*; *EC* = Eutocius' commentary on Archimedes; H = J. Heiberg's edition of Archimedes' works, with Latin translation (*Archimedis opera omnia, cum commentariis Eutocii* [Leipzig, 1910–15; reprint, Stuttgart, 1972]); M = C. Mugler's translation (*Archimède*, tome 1 [Paris, 1970]); Ntr = Netz's translation of *SC* and *EC* (*The Works of Archimedes*, vol. 1 [Cambridge, 2004]). Unless specified, the translation of the Greek is my own. Reference to al-Khayyam is to R. Rashed and B. Vahabzadeh, eds., *Al-Khayyam mathématicien* (Paris, 1999).

2. H 1.186.15–17; see also M 111; and Ntr 202.

3. Archimedes' reduction stretches to four pages: H 1.186.18–190.22. It is the main part of Archimedes' solution to *SC* 2.4 and is later completed by a synthesis of the problem.

4. This is a paraphrase of Archimedes' text in H 1.190.22–25; see also N., p. 14 and Ntr 204. As indicated, I label the problem stated in *SC* 2.4 as Pb1, and this one as Pb2. If the given line and area are specified according to the constraints of the initial Pb1 this gives a third problem Pb2*, which Archimedes states immediately after Pb2 at H 1.190.29–192.5.

5. A reminder will be useful here, if only because it is not found in N.'s book earlier than p. 71—and even there in a partial manner: Eutocius' account of Archimedes' solution is explicitly presented as a reconstitution of Archimedes' missing argument, on the basis of an old manuscript that Eutocius says he recovered with pain. He attributes the manuscript to Archimedes, and the main reason for this is his reconstitution of the argument he goes on to present (H 3.130.29–132.18, mistranslated in Ntr 318, as we shall see). The reconstitution may be divided, for the sake of clarity, into five sections. S1 (section 1) begins

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to comment on the geometrical nature of this solution (part 1.3), by reconstituting Archimedes' line of thought on the basis of a partial translation of the analysis with which Eutocius begins his own reconstitution.[6] In parts 1.4 and 1.5, N. translates the two other solutions of Pb1 that are found in *EC* and comments upon them. The first is that of Dionysodorus, and does not rely on any previous reduction.[7] The second one is Diocles' and it is, like Archimedes' solution, based on a clever reduction, which is much shorter than Archimedes' and leads Diocles to a third problem (Pb3) that is slightly more general than Pb1. Diocles solves Pb3 by an ingenious intersection of conic sections.[8] It is translated again and commented upon in part 1.5. This nest of problems and their solutions, which have in common their being derived from Pb1, are thus successively presented and analyzed by N., who finally describes them as parts of one and the same world, the "World of Geometrical Problems" (part 1.6). In this world, problems are essentially dependent on "the particular terms of a particular geometrical configuration" (p. 57), in contrast to equations, which are (for N.) problems put in some "canonical representation" (ibid.) so as to make their solution identifiable within a classification, such as those one can find in al-Khayyam's *Algebra* or already in commentators of late antiquity like Eutocius. This "essential particularity" of Greek problems is in turn explained within a context (that of early Hellenistic mathematics) pervaded by a deeply entrenched "agonism,"[9] that is, by a generalized endeavor to aim at originality and self-distinction against competitors. Thus, Diocles' commentary on Archimedes is viewed as the "tip of the iceberg," the natural illustration of an all-pervading self-promotion based on harsh criticism. This gives these problems, and their solution, their special "aura" (in Walter Benjamin's sense) and "a sense of uniqueness that defies subsumption under any general heading" (pp. 58–59).

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with an analysis of Pb2 (H 3.132.19–136.13), then S2 continues with the synthesis of the same problem (H 3.136.14–140.20). Then comes S3, a demonstration of the *diorismos* (condition of solubility) to which Eutocius alludes in S1 (H 3.140.21). Finally, in S4, Eutocius adapts his general analysis and synthesis (S1 and S2) to the particular problem equivalent to Pb1 (H 3.150.1–22), thus showing that his reconstitution does fit Archimedes' own words. S5 adds a further proof that the reconstitution is valid: the fact that in his statement of Pb2* Archimedes says we should cut ΔB, and not ΔZ, at X, thus showing that he is aware that the problem can have two solutions when it is soluble (H 3.150.22–152.14). The passage translated by N. in part 1.2 is thus our S2.

6. On p. 22 N. translates only the first part of S1.

7. Dionysodorus' solution (H 3.152.27–160.2) is introduced in *EC* by an important statement (H 3.152.15–26), according to which Dionysodorus produced a new solution of "the whole problem" (Pb1) because he could not read what Archimedes had postponed at the end of *SC*, nor was he capable of finding by himself the missing proofs. Nevertheless, adds Eutocius, he found another, elegant way of invention (*tropon tes heureseos*, not "method of solution," as Ntr 330 has it; see M 101, which is closer to the Greek), that Eutocius goes on to present. See also Eutocius' general introduction in H 3.130.19–22, in which Eutocius compares Dionysodorus' attempt to his own.

8. Pb3 is stated in *EC* at H 162.17–24 (N., p. 41 = Ntr 335). It is also introduced by a *prooimion* (H 3.160.7–162.17), which is reported by Eutocius on the basis of Diocles' *Peri purion* (*On Burning Mirrors*, henceforth *PP*). Both Diocles' *prooimion* and solution are also known through another (and different) version, included in the Arabic translation of *PP*, of which we now have two editions with translations, into English by G. L. Toomer (*Diocles: "On Burning Mirrors"* [Berlin and New York, 1976], henceforth T) and into French by R. Rashed (in *Les catoptriciens grecs*, vol. 1 [Paris, 2002], 119–25, henceforth R). For the Arabic formulation and labeling of Pb3 see T 80–81 and R 122.

9. Following the general views expressed in G. E. R. Lloyd, *Adversaries and Authorities* (Cambridge, 1996). Note, however, that, unlike Lloyd, N. takes his example of agonistic mathematics from early Hellenistic mathematics (see p. 61). For him, the polemics of late antiquity are of a fundamentally different kind from those of previous times (see p. 121, his account of Pappus' tone in his *Collection*).

The first part is naturally followed by two other discussions: the analysis of Eutocius' commentary on *SC* 2.4 (part 2) and the discussion of al-Khayyam's solution of Pb1, which al-Khayyam reduced to one of the canonical problems explained and solved in his *Algebra* (part 3). The tripartite discussion thus defines in outline the general project of N.'s book: to follow up the progressive transformation of a particular problem into an equation,[10] the latter being part of a general classification of problems of the same type.[11] This process defines a historical path, along which each of the different contributions is examined and situated. N. makes it clear from the beginning that his narrative is meant to refute, to some extent, Jacob Klein's view of a deep *conceptual* divide between ancient and modern thought,[12] by admitting (and re-explaining) that there is indeed such a divide, but that it is not essentially conceptual: on the contrary, N. wants to demonstrate that the two are related by a continuous transformation of mathematical *practice*, evolving over centuries, upon which our modern view of such problems is based.[13]

N.'s discussion of Eutocius' commentary on *SC* 2.4 (part 2) is based on the examination of a unique text: the discussion of the conditions of solubility (*diorismos*) of Pb2.[14] N.'s interesting analysis of this crucial text draws mainly from his 1999 article on the same topic,[15] therefore I will only sum up here the main conclusions of his study and point to the addenda to the 1999 article. N.'s first point concerns the attribution of this text to Archimedes. The important question here is whether the entire text is Eutocius' forgery, or whether it can be entirely considered Archimedes' text with a few later emendations. Heiberg, in his 1915 edition of Eutocius' text, had already chosen a middle route, by attributing the end of S3 to Eutocius and therefore holding the beginning to be a verbatim quotation of Archimedes.[16] N. takes up the discussion and, while following Eutocius in his attribution of the main part of the text to Archimedes,[17] goes one step further than H by showing (a) that Eutocius' interventions can be detected inside the text (mainly in his explicit references to Apollonius' *Conica*) and (b) that Eutocius' commentary does not begin at H 148.1, but earlier in the text, at H 144.31. Thus, N.'s careful examination of the text shows that the final section of S3 constitutes an original and important addition to Archimedes' text due to Eutocius that consists in the explicit consciousness of the mutual dependency of the two points of solution of Pb2 (in case they exist). Eutocius' approach is then contrasted with N.'s reconstitution of what could have been Archimedes' original reasoning that led him to the *diorismos* (parts 2.3 and 2.4). Moreover, N.

10. Which problem this is is not always clear, as we shall see: it seems to be sometimes Pb1, sometimes Pb2.

11. In modern terms, these are problems reducible to a "third degree" equation. In al-Khayyam's terms, they are reducible to the fifth of the "trinomial" species of equation that are solved through conic sections only (see R 1.11).

12. The reference is to J. Klein, *Greek Mathematical Thought and the Origin of Algebra*, trans. E. Brann (1968; reprint, New York and London, 1992), originally published as *Griechische Logistik und die Entstehung der Algebra* (Berlin, 1936).

13. The discussion of Klein's thesis comes back as a leitmotif throughout the book. See pp. 4–7, 53, 54, 91, 124, 190.

14. This *diorismos* discussion corresponds to S3 and expands on H 3.140.21–148.27. Archimedes only alludes to it at H 3.190.25–27: he says it exists, without saying what it is exactly.

15. "Archimedes Transformed: The Case of a Result Stating a Maximum for a Cubic Equation," *Archive for History of Exact Sciences* 54 (1999): 1–47.

16. Thus H has the following division: H 3.132.19–146.28 = Archimedes, H 3.148 = Eutocius.

17. N., p. 71: "there is nothing to add to Eutocius' argument."

has a long and detailed discussion on the formulaic expression "The ⟨square⟩ on [*apo*] the ⟨line⟩ on [*epi*] the ⟨line⟩" (part 2.5, pp. 97–120), which is repeatedly used in Eutocius' reconstitution, in Archimedes' alternative proof of *SC* 2.8, and in Eutocius' lemma to the same.[18] The important point is that the *epi* formula, as it is used here, is most naturally explained through its arithmetic connotations, although it is (surprisingly) used here in a geometric-demonstrative context; the second interesting fact is Eutocius' use and "generalization" of the expression in the framework of proportion theory.

There are four addenda to the 1999 argumentation (a1–a4). (a1) Concerning Eutocius' attribution of his reconstitution to Archimedes, there is an interesting note evoking Fabio Acerbi's interpretation,[19] according to which S3 could indeed have been entirely forged by Eutocius so that he could make a claim for himself as a commentator (not as a mathematician). (a2) There is an interesting attempt to trace back the use of the arithmetical *epi* expression in a geometrical context to earlier authors in late Hellenistic times (Hero, Ptolemy) and "early Late Antiquity" (Theon, Pappus), using again some useful suggestions of Acerbi. Hero and Ptolemy are discussed on pp. 112–13, and one finds at pp. 115–16 comments on two interesting quotations on a passage of Theon's *Commentary on the Almagest* (378.10–16 Rome) and of Pappus' famous reformulation of the "four line problem" in book seven of his *Collection* (122–23 Jones).[20] (a3) N. also gives a translation and discussion of Eutocius' lemma to the alternative proof to *SC* 2.8 at pp. 117–20. This analysis leads N. to see Eutocius as a proto-algebraist: a crucial step in his lemma "firmly belongs to the world of algebraic equations" (p. 119).[21] (a4) Part 2, like part 1, ends with a more general discussion of "the problem in the world of Eutocius," which mainly presents Eutocius as a "deuteronomic author," following the ideas expressed in N.'s 2001 article in which he introduces the concept of "deuteronomy" (i.e., a text that "essentially depends on other texts") to characterize late antique and early medieval mathematics.[22] a3 and a4, of course, make the transition to part 3, which deals with those later deuteronomic authors whom N. deems to have had genius (unlike poor Eutocius and his peers, p. 123), namely, Arabic mathematicians.

Part 3 begins with an interesting discussion of the fate of Pb1 "in the Arab world" (part 3.1). After a quick discussion of the textual traditions of both Archimedes' works and Eutocius' commentaries in Arabic translations, N. mainly dwells on

18. The corresponding Greek expression is, for example, "*to apo tes AB epi ten AD*," meaning "the solid figure whose base is the square on the line AB and whose height is the line AD," or perhaps, "the square on AB multiplied by the line AD."

19. P. 76, n. 55, citing a private communication from Acerbi.

20. This passage indeed became famous through Descartes's quotation and discussion of it in the two first books of his 1637 *Geometrie*.

21. N. often compares Eutocius' approach to the modern one (using calculus); for example, he argues that Eutocius already has "an explicit concept of functional relation between mathematical objects" (p. 94, repeated at p. 96).

22. "Deuteronomic Texts: Late Antiquity and the History of Mathematics," *Revue d'histoire des mathématiques* 4 (1998): 261–88. Two reactions have been published to this article in the same journal: Karine Chemla's "Commentaires, éditions et autres textes seconds: Quel enjeu pour l'histoire des mathématiques? Réflexions inspirées par la note de Reviel Netz," *Revue d'histoire des mathématiques* 5 (1999): 127–48, and my "Comment définir la nature des textes mathématiques de l'antiquité grecque tardive? Proposition de réforme de la notion de 'textes deutéronomiques,'" *Revue d'histoire des mathématiques* 9 (2003): 101–43. See also Jens Hoyrup's remarks on N.'s 1998 article in *Mathematical Reviews* (2000g:01012); and N.'s response to the two articles, forthcoming in *Revue d'histoire des mathématiques*.

al-Quhi's interesting solution and completion of the problems in *SC* 2.4, apparently following Franz Woepcke's 1851 discussion of it.[23] The contributions of al-Mahani, Abu'l-Jud, al-Khazin, Ibn al-Haytham, are also discussed (pp. 133–37). Al-Khayyam's discussion of the equation to which Pb1 can be reduced is then presented, with two preliminaries: a discussion of al-Khwarizmi's treatise on algebra (part 3.2), which contains in particular a brief discussion of the meaning of "equation," and a general discussion of the structure of al-Khayyam's algebra,[24] with a special emphasis on al-Khayyam's special obsession with classifying everything according to genera and species and on the confusion between the project of an *Algebra* and that of an *introduction* to algebra (part 3.3).[25] N. then gives a free English translation of al-Khayyam's solution of "problem 17" (part 3.4), which allows him to compare it with Archimedes' alleged solution of Pb1 (part 3.5). While al-Khayyam's treatment is shown to be closer to Greek traditional geometry (and hence to Archimedes) than to modern symbolic algebra (pp. 161–65), N. points out two divergences between al-Khayyam and Archimedes, namely the central role played in al-Khayyam by the study of cases (whereas Archimedes shows little interest for it) and the preeminence of manipulations of equalities over proportions in al-Khayyam, contrary to Archimedes' practice. Part 3 continues with a discussion of al-Khayyam's explicit criticism of Abu'l-Jud's solution of "problem 17" and the style of al-Khayyam as a polemist ("as compared to the Greeks"). N. also dwells on Sharaf al-Din al-Tusi's renewed treatment of al-Khayyam's classification according to new criteria (pp. 178–81), and ends up with general remarks on the transformation of Archimedes' original problem into an equation (i.e., part of a classification) by Arabic mathematicians.

I am not entirely convinced by N.'s account of both Archimedes and Eutocius. First, as the above summary of the textual situation should make clear, we have in *SC* three problems (Pb1, Pb2 and Pb2*), which are unambiguously distinguished from each other, in the Greek text, by their different *protaseis* or "statements." By contrast, N.'s presentation of them is much looser; in fact, he seems to speak about "*the* problem" exactly as he would speak about "*the* difficulty."[26] This tendency is explained by N.'s essential project in part 1, which is to try to follow the heuristic

23. Woepcke, in his *L'Algèbre d'Omar Alkhayyami* (Paris, 1851), has indeed a long discussion of al-Quhi's work, which is briefly mentioned by al-Khayyam at 196 and 254. Woepcke has also a long excursus ("addition C") on al-Quhi's contribution, which seems to be taken from Nasir al-Din al-Tusi's "reworking" (*Tahrir*), written toward the end of the thirteenth century C.E. An updated edition of it, with translation and commentary, has been provided by L. Berggren, "Al-Kuhi's 'Filling a Lacuna in Book II of Archimedes' in the Version of Nasir al-Din al-Tusi," *Centaurus* 38 (1996): 140–207.

24. For this N. relies on al-Khayyam (the *Algebra* and the related treatise *On the Division of a Quadrant of a Circle*). It should be noted, however, that this edition of the *Algebra* (see n. 1 above) is basically the (almost) verbatim reproduction of A. Djebbar and R. Rashed, *L'œuvre algébrique d'al-Khayyam* (Aleppo, 1981).

25. N. indeed argues that both projects are essentially one and the same thing for al-Khayyam.

26. See, for example, N.'s comments on Dionysodorus' strategy: [he decided] "to solve *the* problem in the particular terms of the cutting of a sphere" (p. 37, my emphasis), as if there were in all this only *one* problem, whereas there are obviously (and, I would say, fundamentally) *many* problems. In the case of Dionysodorus, the confusion is further complicated by the fact that N.'s analysis relies on two mistakes: first, N. asserts that the idea that line ZA is equal to the radius of the circle AπB (p. 36; see figure in H 155 or N., p. 31) plays no part in Dionysodorus' reasoning, which is just false, since it enables Dionysodorus to use the equality he proves in his lemma at H 3.158.13–160.2 (p. 32, n. 61 betrays the fact that N. did not see the importance of the lemma); secondly, N. analyzes his solution as if it were directed toward a problem that is stated nowhere by Dionysodorus as such (p. 33).

line of thought that could lead Archimedes, Dionysodorus, and Diocles to their particular solution of Pb1. But this special emphasis on mathematical heuristics also leads N. to obliterate the specificity of the ancient method of analysis as a means of solving problems. Indeed, the clever and progressive shift from one problem to another is the gist of ancient methods for solving problems. This method consisted for the most part in the manipulation of auxiliary diagrams,[27] the transformation of proportions, or the use of the properties of special curves, like the sections of the cone. The goal was to arrive, through these geometric manipulations,[28] at problems that were already "known" (in the sense that solutions were known), or problems that *looked like* "solvable" problems. Thus, to take an early example, the Delian problem would lead to the problem of inserting two mean proportionals between two given lines, a problem which is close to that of inserting one mean proportional. In the same way, Archimedes' reduction leads one from Pb1 to Pb2, a problem of section that looks solvable.[29] Hence, Hellenistic geometers had their own ways of solving problems, by a method of reduction that was not entirely standardized (thus leaving much room for innovation and, ultimately, for standardization) but that was still an efficient auxiliary to the mathematical process of invention.[30] By ignoring this general context, that is, the "topical" reference to canonic problems or techniques, N. is led to represent the solutions of Archimedes, Dionysodorus, and Diocles as if they were surrounded by a romantic "aura" that would give them their uniqueness. But this presentation could be applied to any demonstration of a problem: just forget about the topical context, and you will inevitably discover that each demonstration had its personal "aura."[31] This omission also makes it difficult to understand how Arabic mathematicians might have been interested in comparing the Greek geometrical method of reduction with the method of *jabr* and *muqabala* that was first explained in a coherent treatise by al-Khwarizmi at the beginning of the ninth century C.E.[32] I suspect that N. was misled by the particular form of al-Khayyam's algebra. Al-Khayyam could indeed do away with the explanations about the rules of algebraic calculation, since he came at a time when this technique was already well established.[33]

27. Like those found in Book 2 of Euclid's *Elements*.

28. Manipulations of proportions—unlike what N. suggests in part 1 in a very confusing manner (for example at p. 53)—are part of geometrical practice and have nothing "quantitative" in themselves. Generally speaking, a lot of things are called "algebraic" in the book that have in fact little to do with algebra.

29. Just imagine Pb2 without the squares: to cut a given line ΔZ at X such that the one segment ΔX is to a given line as another given line is to the other segment XZ. This one would be easily solved by a simple application of areas, one of the "canonical" problems contained in Euclid's *Elements*.

30. For an insightful discussion of the ancient method of analysis, M. Mahoney, "Another Look at Greek Mathematical Analysis," *Archive for History of Exact Sciences* 5 (1968): 319–48, is still valuable. N. has a general argument about the role of published *analyseis* in the Greek world according to which they have little to do with heuristics (see his "Why Did Greek Mathematicians Publish Their Analyses?" in *Ancient and Medieval Traditions in the Exact Sciences: Essays in Memory of Wilbur Knorr*, ed. P. Suppes, J. M. Moravcsik, and H. Mendell [Stanford, 2000]). I have difficulty in reconciling this strange view with N.'s own use of extant *analyseis* for the reconstitution of ancient heuristics.

31. In this respect I must confess that I do not really see why one could not uncover al-Khayyam's "mode of discovery" (p. 161); the remark is all the more paradoxical in that al-Khayyam does all that he can to develop heuristic thinking in his reader (see for example al-Khayyam 238–42).

32. It is probably significant that, commenting on al-Khwarizmi (pp. 139–40), N. makes only very brief remarks on algebra as a method of reduction, as well as on the foundation of algebraic calculation, which actually forms the core of al-Khwarizmi's treatise.

33. See the remarks of R. Rashed in *Histoire des sciences arabes* (Paris, 1997), 2:44 on the difference between al-Khayyam's project and the trend previously represented by the "algebraists-arithmeticians."

He could thus concentrate on the solutions of particular equations, since the method of reduction itself was no longer a challenge as it had been for Greek mathematicians.

The second difficulty I have with N.'s account is in his treatment of Eutocius' project. I find it somewhat artificial to focus on one short stretch of text when dealing with Eutocius' contribution, when all the texts used and translated in part 1 come from Eutocius' account. On this point I can only repeat what I have already explained about N.'s 1999 misleading account of Eutocius' project:[34] contrary to what N. says on p. 72, Eutocius nowhere promises to *transcribe* the text he has found; furthermore, N. says that "[Eutocius] will obliterate the very reasons that made him think this was by Archimedes, i.e. he will rewrite the proof with modern terminology" (ibid.), which is just the opposite of what Eutocius actually says, namely, that he has first *studied* with most care the text, "as it has been written," so as to be able to find out the main ideas (*ennoia*) from it and put it in suitable (i.e., "modern") language (*lexis*).[35] In other words, Eutocius' fundamental project, which is nowhere really analyzed by N., although Eutocius makes it quite explicit, is to build a *mathematically coherent* discussion on the basis of the corrupted text he has found. It should be coherent both in itself and with Archimedes' statements about Pb2, so that Eutocius may convince his readers (and perhaps himself) that the text he has found should indeed be attributed to Archimedes. Thus, Eutocius' meaning is clearly not to obliterate, but, on the contrary, to focus on his strongest argument in favor of the attribution.[36] Generally speaking, the lack of discussion of Eutocius' project leads N. to adopt without enough discussion crucial conclusions and ideas, such as the attribution to Archimedes of the proof Eutocius corrected, or the idea that Dionysodorus created his solution of Pb1 because he did not want to appear second to Archimedes.[37] More subtly perhaps, N. shares with Eutocius (and with many other late antique authors, like Pappus) a special and characteristic obsession for invention (*heuresis*) and problem-solving activities.[38] Like him, he is fond of reconstituting what Eutocius nicely calls the "way of invention" (*tropos heureseos*) of the Ancients.[39] This is certainly interesting, and a great deal of speculative ability is required to imagine the heuristic reasoning plausibly followed by the ancients, much in same the way as Wilbur Knorr enjoyed doing before N., but it hardly amounts to a critical and contextual study of Eutocius' practice and project.

34. See my article "Comment définir" (note 22 above). N.'s recent translation of Eutocius' introduction (Ntr 318) is equally misleading; M 89 has a correct translation.

35. This is of course a completely different, and actually much more subtle idea than mere transcription. Note that the *ennoia/lexis* distinction is standard in ancient rhetoric.

36. Incidentally, this also confirms, on the basis of simple arguments of context, that the final part of S3 is indeed by Eutocius: he later explains that the existence of two points may explain Archimedes' special formulation of Pb2*.

37. There is a similar problem regarding Diocles' solution of Pb1: Eutocius, probably because he had reconstructed a text that could be Archimedes' solution, insisted that Diocles had invented his own solution of Pb1 after reproaching Archimedes with having not fulfilled his promise. But Diocles' own words show that his main criticism was not the lack of a demonstration, as N. suggests following Eutocius, but the length of Archimedes' reduction; Diocles' own reduction was much shorter, and therefore elegant. Moreover, the fact that Archimedes mentions the *diorismos* in *SC* would make it clear to any reader that he had found a solution for Pb2.

38. This obsession is already manifest in W. Knorr's *The Ancient Tradition of Geometric Problems* (Cambridge, Mass., 1986).

39. H 3.152.20–21, which, for no good reason, is mistranslated as "the method of solution" in Ntr 330. Following Rashed's clever remark in R 19, much of what Eutocius says is explicitly presented as a pastiche, *à la manière de*.

Apart from the important problems mentioned above, I was surprised not to find any discussion of Diocles' text based on a comparison between Eutocius' version and the Arabic translation, although the two versions differ significantly at several places, and each of them appears to complete the other. Moreover, it seems to me that such a comparison would have fit N.'s general argument quite well.[40] Equally surprising is the lack of any reference to the textual tradition between Archimedes and Eutocius: pp. 112–15 is a good beginning, but much more could be said, given the famous discussion of Eutocius on compound ratios (for which there seems to be a lost Nicomachean source)[41] and Hero's quotations of *SC*, which seem to be among the rare attestations of the intermediary tradition.[42] It is, generally speaking, strange to read the suggestion that nothing really happened between Archimedes and Eutocius (p. 131), whereas Eutocius himself makes it clear that he had not been the sole commentator on Archimedes' books; he certainly pretended to be better than his predecessors, but not that there had been none.[43] Finally, on the (obviously important) question of the translations of Archimedes and Eutocius into Arabic, I was struck by the fact that N. (pp. 130–31) seems to arrive at conclusions that significantly differ from those of Richard Lorch,[44] which should have called for some comments. Generally speaking, an examination of the Arabic translations of Eutocius would have been appropriate in such a study.

Concerning N.'s general agenda about the "deuteronomic nature" of commentaries from late antiquity and Arabic writings, I must confess that I cannot see why Diocles' account should be regarded as less "deuteronomic" than the texts from late antiquity. For what is his solution, if not a very clever commentary on Archimedes' own words, in order to show that one can arrive much more quickly at a solution of Pb1 than Archimedes did? How can one say that Diocles' solution is independent of Archimedes' (p. 160), when his basic meaning is that Archimedes already had, without knowing it, a reduction in his hands? Diocles' solution is, on the contrary, brilliantly dependent upon that of Archimedes. This shows, not surprisingly, that Diocles had his own classics and that he had read them with the greatest care.[45] Generally speaking, it is not a good sign for any general agenda to have to distort the facts and the sources to make them comply with it; if this is the case, something must be wrong with the general idea.[46]

40. N. (p. 95) shows he is aware of this necessary comparison, but his translation itself has no trace of it; by contrast, Toomer and Rashed both tried to make the comparison in the notes to their translations of Diocles (see n. 8 above).

41. Cf. H 3.120.20–23; and Knorr's commentary in his *Textual Studies in Ancient and Medieval Geometry* (Cambridge, Mass., 1989), p. 170, n. 17.

42. Ironically, N. does not seem aware that Unguru, whom he criticized for not having given any truly historical account of the difference between ancient and modern thinking (p. 4), recently tackled just this problem in the first chapter of M. Fried's and S. Unguru's recent study of Apollonius of Perga's *Conica* (Leiden and Boston, 2001), pp. 20–22. On Hero's contribution to the "rewriting" of Euclid's Book 2, see F. Acerbi, "Lesser Hero: Forms of Analysis and Synthesis in the Ancient Corpus" (forthcoming).

43. On this point N.'s study should be compared with Micheline Decorps-Foulquier's recent and more careful account of Eutocius' activity: *Recherches sur "les Coniques" d'Apollonios de Pergé et leurs commentateurs grecs* (Paris, 2000).

44. R. Lorch, "The Arabic Transmission of Archimedes' *Sphere and Cylinder* and Eutocius' commentary," *Zeitschrift für Geschichte der Arabische-Islamischen Wissenschaften* 5 (1989): 94–114. I quote from p. 108: ". . . the commentaries to II 1 and II 4 were clearly fairly well-known . . ."

45. Incidentally, this is probably one of the major reasons why Eutocius quoted Diocles' *prooimion*: he had found an early piece of commentary of one mathematician upon another.

46. For more detailed criticisms on this point, see the articles quoted in n. 22 above.

To sum up, N.'s study certainly shows much ambition, but it is not always reliable. It too often distorts (when it does not just ignore) the primary and secondary literature, and it is based on a very questionable representation of the ancient method for solving problems. It should not be recommended to anyone wishing to acquire precise knowledge of the complex questions underlying the transformation of early Hellenistic mathematics in the hands of late antique and medieval commentators and mathematicians.[47] Nevertheless, it must be said that, with all its deficiencies, the book is very rich and contains many stimulating and original ideas that call for fruitful reflection. It also constitutes a rare attempt to provide an overview on a very difficult topic. As such, and despite its flaws, readers who already have a strong command of the questions and texts mentioned above, and who are looking for new and stimulating ideas, will certainly find N.'s book a useful contribution.[48]

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47. For this, they should rather read Knorr's seminal *Textual Studies* (note 41 above), which, for some extraordinary reason, is not quoted anywhere in N.'s book. Generally speaking, the poverty of bibliographic references of the book is amazing: only one page and a half of reference, when any single article on late antiquity has at least twice this amount! Moreover, the few studies that are cited are too often misquoted and misinterpreted (including those by Unguru, Heath, Mansfeld, Klein).

48. I thank all the colleagues whose many helpful remarks assisted me in producing the final version of this review. The reader who might be interested in a more detailed review of the same book, including an alternative interpretation of the late antique texts examined by Reviel Netz in the second part, is encouraged to read Fabio Acerbi's paper, "Archimedes and the Angel," to be published online in *Aestimatio*, http://www.ircps.org/publications/aestimatio/aestimatio.htm.